Functions & calculus - Solutions

1 Functions



|  |  |  |
| --- | --- | --- |
| Function | Domain | Range |
| (b) f(x) = x1/2 | [0, ∞) | (-∞,∞) |
| (c) f(x) = 1/x | (-∞,∞) & x0 | (-∞,∞) & f(x) 0 |
| (d) f(x) = loge(x) | (0,∞) | (-∞,∞) |
| (e) f(x) = exp(x) | (-∞,∞) | (0,∞) |
| (f) f(x) = |x| | (-∞,∞) | [0,∞) |
| (g) f(x) = x2 | (-∞,∞) | [0,∞) |

* 1. To find the roots of the quadratic equations we made either factorise or use the following formula:

where ax2 + bx + c = 0.

1. *x*2 + *x* – 6 = (*x* + 3)(*x* - 2) = 0, so the roots are -3 and 2.  
     
   Alternatively:  
     
   .
2. -3*x*2 - 2*x* + 1 = (-3*x* + 1)(*x* + 1) = 0, so the roots are -1 and 1/3.  
     
   Alternatively:  
     
   .
3. *x*2 – x + 1 does not factorise.  
     
   Alternatively:  
     
   . So no real roots.  
     
   What is going on here? Sketch the function if it is not immediately apparent.

1.3 a0 = 1

32 = 9

13 = 1

2-3 = 1/8

43/45 = 16-1

(3-3)3 = 3-9

52/32 = (5/3)2

43 = 26

27-2/3 = 1/9

1.4 log10(10n) = n

log10(104/10-3) = 7

1/2log10(16) = log10(4)

log10[log10(10)] = 0

log10(1000)/log10(100) = 3/2

1.5



2 Differentiation

2.1

1. Differentiate the following functions:

|  |  |
| --- | --- |
| Function | df(x)/dx |
| f(x) = x3 | 3x2 |
| f(x) = x-7 | -7x-8 |
| f(x) = | x / |
| f(x) = 1/loge(x) | -1/[x(loge(x))2] |
| f(x) = exp(4x) | 4exp(4x) |
| f(x) = (1+x)/x2 | (x2 - 2x(1+x))/x4 |

2.2 (a) F’(x) = 8-2/x2 =0 ⇒ x = ± ½. F”(x) = 4/x3.

At x=-1/2, F”(x) = 4/x3 = -32 which is < 0, thus F(x) has a maximum at x = -1/2.

At x=1/2, F”(x) = 4/x3 = 32 which is > 0, thus F(x) has a minimum at x = 1/2.

(b) F’(x) = -3x2 - 2x + 1 = 0 ⇒ x = -1 or 1/3 (as per 1.2 (b)). F”(x) = -6x - 2.

At x=-1, F”(x) = 4 which is > 0, thus F(x) has a minimum at x = -1.

At x=1/3, F”(x) = -4 which is < 0, thus F(x) has a maximum at x = 1/3.

2.3 (a)  (b) 

(c)  (d) 

(e)  (f) 

3 Integration

3.1  3x5 /5 + *c* 3.2

3.3 loge (x2 + 4) = loge + *c*

3.4 (a) (i) (ii)

(b) (i) (ii)

4 Further differentiation

4.1

4.2

4.3

4.4

4.5 2

4.6 4*σ*

5 Finding a maximum likelihood estimate

i) .

ii) *l*(*σ*2) = log(L) = (omitting terms not in *σ*2).

iii) *l’*(*σ*2) =

iv) MLE(*σ*2) =